

SOME PHILOSOPHICAL AND SOCIAL SIGNIFICATIONS OF SETS IN LOGIC

MANTIK İÇERİSİNDE BAZI FELSEFİK VE SOSYAL ANLAMLAR

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Abstract

This paper discusses the philosophical and social significations of sets in three logical systems; two-valued logic, many-valued logic, and fuzzy logic. Here is an attempt to infer these significations from the definition of sets; graphically represented by Venn diagrams or membership functions, and the basic operations on sets. This paper explains that each set is a reflection, either explicitly or implicitly, of human thinking's ways. Two-valued set as a representation of binary and strict thinking, many-valued set as a representation of multiple thinking, which is less strict than binary thinking, but also less open than fuzzy or open and freethinking, which is represented by fuzzy set. Unlike those who say that the one, but not the other, of these three logical systems can represent real world, and reflect human thinking's ways, this paper detects that each one of these systems has more suitable scope than the other, and can reflect one way of thinking better than the other does. Therefore, each logical system has its importance and role in real life, and has scope that is more appropriate to represent it. Finally, it asserts that logical systems do not exhaust human beings' life.

Keywords: Sets -membership values- basic operations, fuzzy logic.

Özet

Bu çalışmanın amacı mantık sistemi içerisinde yer alan iki değerli mantık, çok değerli mantık ve bulanık mantığın sosyal ve felsefik anlamlarını tartışmaktır. Bu anlamları tartışmak amacıyla VENN diyagramlarından, grafiksel fonksiyonlardan ve bazı temel işlemlerden faydalanılmıştır. Makalenin bulguları dikkate alındığında her bir mantık setinin insan düşüncesinin açık ya da örtülü bir yansıması olduğu görülecektir. İki değerli mantık seti ikili ve sıkı düşünmeyi, çoklu mantık seti ise çoklu düşünmenin ikili düşünmeden biraz daha farklı olduğunu belirtmiştir. Serbest düşünme ya da açık veya açıktan daha az düşünme bulanık mantıkla ifade edilir. Mantık sistemi az ya da çok gerçek dünyayı gösterir, insanın düşünce dünyasını yansıtır. Bu makalede mantık sistemlerinin her birinin diğerinde daha uygun olduğu tespit edilmiş, düşüncesinin bir yönünün diğerlerine göre daha iyi yansıttığını tespit edilmiştir. Bu yüzden mantık sistemi hayatımızda büyük öneme sahiptir. Sonuç olarak mantık sistemi insanların hayatlarının bir dumandan ibaret olmadığını ifade eder.

Anahtar Kelimeler: Veri setleri, üyelerin değerleri, esas işlemler, bulanık mantık

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Introduction

Long time ago, logic was concerned with the truth and falsity of propositions. That concern was, and is still, a reflection of different ways of thinking; i.e. logic is not a mere judgment of propositions' truth and falsity, rather it is an implicitly vision or point of view of reality and human being. Such vision is represented in three ways of thinking. The first one is binary or definite and strict thinking that represents two-valued logic; the second one is multiplicity that represents many-valued logic, which is indefinite and less strict than the binary thinking; and the third one is free and open thinking that represents fuzzy logic.

The components^(*) of those three kinds of logic reflect, each by its own way, different ways of thinking, and a stance on real life and human affairs. Either implicitly or explicitly, each component is formed of strict, indefinite, or freethinking. Here is an attempt to concentrate on some philosophical and social significations only of the sets in the three logical systems. Therefore, we will focus on the sets' definition, the representation by Venn diagrams or membership functions, and the basic operations on crisp- and fuzzy sets only; because logicians do not take into consideration the basic operations on Many-valued set.^(**)

Sets' Definition and its significations:

The sets definition depends on the classification of elements according to their belonging. In classical sets; i.e. crisp or Two-valued sets and many-valued sets, this classification is different from the classification in non-classical ones; i.e. infinite or fuzzy sets. In Two-valued set, the classification of elements is too strict, that means, there is no intermediate belonging. While in many-valued set, there are some partial degrees of belonging or membership. By contrast, in fuzzy sets there are infinite values of membership between full membership and non-membership.

According to crisp set, each element has only two possibilities; either it belongs to the set or does not belong. (Yaon Klir: *Fuzzy Sets and Fuzzy Logic*, p. 6.) In other words, the element is either fully in or fully out the set. Hence, the membership function can take only two values, one and zero.

Since the classification of two-valued set is not sufficient to represent all objects in real world, many-valued set adds intermediate values between fully in and fully out the set. These values are denoted by neutral, indeterminate, or undefined. Therefore, three-valued set substitutes the intermediate value with single number;

^(*)I mean by logic components; the terms, the propositions, and the inferences, or the membership functions of sets and the basic operations on sets.

^(**) The main subject which Logicians focus on, when they have been dealing with Many-valued logic, was to find truth values of propositions that are neither true nor false; i.e. undetermined or undefined truth-values. Therefore, they did not take into consideration the membership functions or the basic operations on these sets. Furthermore, conjunction, union, equality of two or more propositions that have undetermined truth values, particularly (0.5), were studied. Logicians also highlighted on the implication of propositions, which could be inferred within more than ten ways. Through the implication function studying, we could see many points of view that mean logicians have resulted different consequences. Thus, the basic operations on those sets have differed also. Even though, we could not find such operations on Many-valued sets, rather logicians have only compared those operations on Two-valued and fuzzy sets. Some of them believed that the basic operations on fuzzy sets are an extension to those in Two-valued sets, while others believed the basic operations on Two-valued sets are special cases of those on fuzzy sets. C. f., Merrie Bergmann: *An Introduction to Many-Valued, and Fuzzy Logic: semantic, algebra, and derivation system* & Graham Priest: *An Introduction to Non-Classical Logic: from if to is* & George Bojadziev; Maria Bojadziev: *Fuzzy Logic for Business, Finance, and Management* & Gorge Klir; Bo Yuan: *Fuzzy Sets and Fuzzy Logic: theory and application* & Sivanandam and others: *Introduction to Fuzzy Logic using MATLAB*.

four-valued set substitute sit with two values; five-valued set substitutes it with three values, and so on. (Bart Kosko: *Fuzzy Thinking: The New Science of Fuzzy Logic*, p. 19.)

Suppose that some elements have fully membership value (1) or fully non-membership value (0), then the other elements would have values between these two. Therefore, the membership values of two-valued set are $\{0, 1\}$, of three-valued set are $\{0, \frac{1}{2}, 1\}$, of four-valued set are $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$, of five-valued set are $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$, and of six-valued set are $\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\}$.

On the contrary, in fuzzy set, the elements have infinite membership values between fully in and fully out, and the transition between any two values is continuous and smooth. Fuzzy set, as defined by Zadeh, is a "class" with a continuum grade of membership, or it is characterized by a membership function, which associates with each point in the space of objects as a real number in the interval $[0, 1]$. (Lotfi Zadeh: "Fuzzy Sets", *Information and Control* 8, p. 339)

Related to representations of reality, the classification of each set reflects some real facts better than the other set does. According to two-valued set, classification reflects the specific and clear facts; i.e. the facts that do not have any borderline cases, such as the set of "permanent members of the security council". According to many-valued set, classification reflects the undetermined facts, such as the set of "Fourth-year students of the department of philosophy at Damascus University"^(*). While classification in fuzzy set, reflects vague facts; i.e. facts that have borderline cases, or facts that are difficult to draw a sharp line between its cases, such as the set of "young" people.

In this sense, we may notice that each set can reflect an aspect of reality, or can represent this or that case, better than the other set does. This means that it is more appropriate to take into consideration phenomena's characteristics as much as possible by using the appropriate set when we classify cases. Therefore, if the two-valued set is used with vague phenomena –as trying to classify all objects, strictly and definitely, according to the binary way of thinking-this may distort these phenomena and reduce them into separated objects. Likewise, if the fuzzy set is used with definite or clear cases –as trying to consider all things vague and overlap, exploiting flexibility of fuzziness-that may distort them, or make the mambiguous. However, in most cases, it is difficult to separate between real facts, or draw a sharp line between them.

These classifications reflect also various points of view for different aspects of life. Such as the full membership and the independent absolute truth are closely correlated, which is an attitude adopted by those who have a restricted way of thinking that depends on Two-valued Logic. On the contrary, for fuzzy theorists there is no absolute truth, rather there is a fuzzily defined reality. They consider reality as dependent on our perceptions and the truth as a "matter of degree" (Burhan Türksen: *an Ontological and Epistemological Perspective of Fuzzy Set Theory*, p. 10.)The first attitude reflects reductionism and absolutism, whereas the second one reflects anti-reductionism and relativism.

In order to classify cases strictly, we must reduce multifaceted qualities by imposing a kind of homogeneity with the absolute truth, and a kind of contradiction between this truth and anything different from it. Thus, according to this point of view, the

^(*) According to Damascus University's rules, some students succeed in all subject, others fail in some subjects, others have promote to upper class in the first semester, and take two subjects from the fourth-year. All of them are in the set of "fourth-year student", but each of them has different membership value.

existence converted into essence; the knowledge converted into correspondence to reality; and the truth converted into absoluteness. While, the fuzzy point of view considers reality as heterogeneous and diverse, truth as relative, partial, and variable.

Consequently, these stances of truth and reality lead to the emergence of many judgments in human being's life and in natural science fields. In human beings' life, absolute truth is manifested in different types of power, to higher or lesser degree. It is manifested, in the religious field through priest power; in the political field through totalitarianism, and in the social field through patriarch power, in the masculine society, and through matriarch power in the feminine society. In natural sciences, absolute truth is manifested in many concepts; such as determinism as leading to implicitly metaphysical stance; objectivity as picturing mind as precise as mirror of nature; and rationality as one-sided stance that leads to exclusionism.

As for multiplicity thinking which is based on many-valued logic, it represents less strictness and less reduction than binary thinking, but less flexible than fuzzy thinking because multiplicity thinking allows some intermediate values to take place. While reductionism still has its impact through multiplicity thinking, it becomes at its lowest level in fuzzy thinking.

Historically, these three kinds of belonging; i.e. binary, multiple, and fuzzy, which are manifested in human thinking, emerged as the concept of individuality or ego, at the ontological level; and as certainty and exchanging philosophical and scientific experiences at the epistemological level.

According to binary thinking, there is only one person who possesses individuality, free ego, and has certain knowledge. He is the ruler who has the absolute truth and the absolute authority. This way of thinking arises in the oriental ancient mind. While in multiplicity thinking, some people possess individuality and free ego. This way arises in the Greek era through the social hierarchical classes.

Comparing to this and that way of thinking, in fuzzy thinking, all people exist, to higher or lesser degree, and all have an important role, each of them chooses his role freely that arises in democratic societies.

Nevertheless, these ways of thinking do not mean that there is identity between those logical systems on the one hand, and these ways of thinking and their ontological, epistemological, and social significations on the other hand. Human beings are thinking either strictly multiply or fuzzily, before the logical systems have formulated, as well as the social significations in these systems do not appear completely in such distinction way.

Thus, if those classifications for each set reflect the different way of thinking, therefore graphically, either Venn diagrams or membership functions will make these representations clearer.

Membership functions and their significations:

Both Venn diagrams and membership functions allow us to highlight many philosophical and social significations of each set. However, we have to remember that two-valued set could be represented in both ways, i.e. Venn diagram and membership function, while many-valued and fuzzy^(*) sets could be represented only by membership function. Therefore, we notice that Venn diagram detects that two-valued set has closed and separated characteristics. We could interpret the closed

^(*) In fuzzy sets, the membership functions has many important roles, such as determination of the universal sets, specifying the number of terms and supporting intervals. However, we concentrate here only on its signification without talking about these roles.

characteristic, in some cases, as strictness and close-minded, and in other cases, as principle decisive account. While separated one may interpret, on the one hand, as simplicity view of point, which draw a sharp line between cases, without considering vagueness, and substitute the whole by its parts, and on the other hand, it may interpret as high precision to select elements.

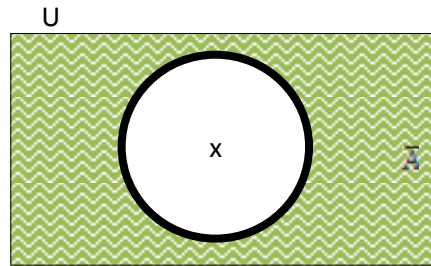


Fig. 1: Venn diagram

As illustrated in fig.1(Venn diagram of two-valued set), we can detect those two characteristics. We notice that only the set's members are fully in the set, while the non-members are fully out the set. Suppose an object (x) belongs to the universal set (U), that means, it is either fully in or fully out the set (A).When it is fully out the set (A) it belongs to the complement set of (A), i.e. (\bar{A}).

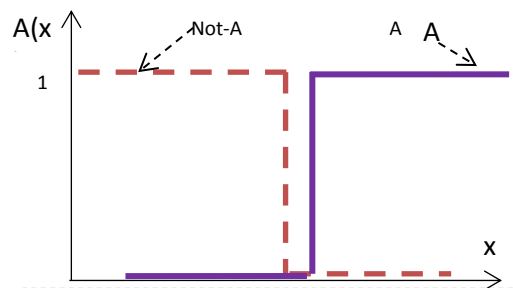


Fig. 2 Membership Function of two-valued set

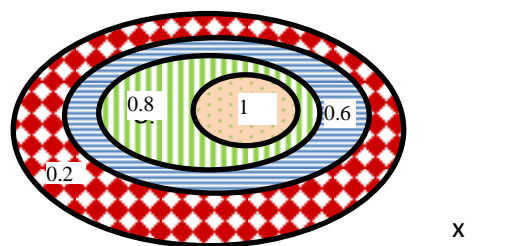
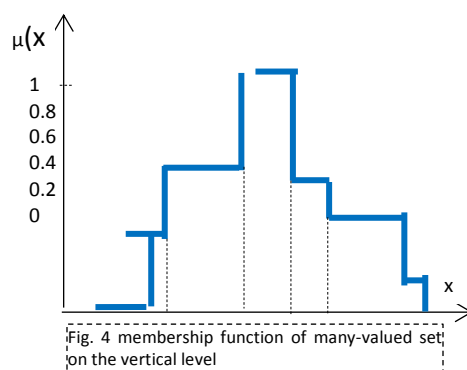


Fig. 3 membership function of many-valued set on the horizontal level

In addition, in some aspects, the membership function of this set maybe interpreted as static structure divided by contradictions into two separated parts. The two parts look as separate as the ideal and real worlds in Plato's philosophy. Thus, they reflect the top-down relationship. As shown in fig.2 (the membership function of two-valued

set), the separation between contradictions is made by a vertical line which is parallel to the membership axis (Y-axis). Members and non-members' relationship is similar to both ends of the line segment's relationship in Euclidean geometry. In such relationship, each object exclude sits opposite. Therefore, the object is either in the top or in the bottom without any gradation between them. According to this membership function, each element transfers drastically and abruptly steps from non-member (zero) to fully-member (one), i.e. the element abruptly becomes in or out the set, or moves drastically from the top to the bottom of membership function, and vice versa. We can recognize this kind of relation in our daily life, such as in someone's unexpected wealth, or in some employee's sudden promotion, as military promotion resulted from succeeded unenviable task, or military coups that turn positions upside down.

On the contrary, the membership function of many-valued set looks like circular structure, on the horizontal level; and looks like hierarchical structure, on the vertical level. As illustrated in fig. 3, the circular structure on the horizontal axis (x-axis) represents multiple circle sat the same level. This multiplicity represents sameness and equality existence of people." They meet at eye level and everyone inhabits the same level [...] without displacing someone." (Linda Jean Shepherd, *Lifting the Veil: The Feminine Face of Science*, p. 139.) This structure could be found broadly in real life indifferent multi faraway groups, such as social, political, or racial ones. In this sense, these kinds of differences would not allow any conversations, or even if there are such conversations, they would be unfruitful.



In addition, the membership function of many-valued set looks like hierarchical structure on the vertical level. As illustrated in fig. 4, the hierarchical structure on the vertical axis (y-axis) represents gradation relationship changing, step-by-step, up-down and vice versa. Such relation looks like the regularly promotion or demotion of employees, i.e. each employee stays at the same position for a while, then those who made fulfillments and hard workings would have regularly promotion, those who made big mistakes such as breaking the rules would have punishment by demoting them to a lower position.

However, Linda Shepherd criticized the bias of science towards hierarchy, simplicity, linear progress, and either/or thinking as a dominance of masculine consciousness. She discussed that hierarchical structure "emerges out of western dualistic either/or thinking, and is based on linear logic- the absolute classification of things.[...] the tendency toward thinking of either/or, i.e. truth or falsity." (*Ibid*, p. 129.)According to her discussion, the hierarchical structure, which is based on binary thinking, reduces multiplicity into unitary thing, converts the complex and subjective

into a single number, fosters competition power struggle, and excludes the other and even abrasive under the excuse of objectivity. (*Ibid*, pp. 124-127.)

I agree, to a certain degree, with Shepherd's discussion of the hierarchical structure, but unlike her viewpoint, I emphasize that two-valued logic does not represent this structure; however, many-valued logic represents it. In fact, the two-valued logic represents static structure, because (as membership function illustrates in fig. 2), it has only two separated levels, the upper, and the lower one. Therefore, the binary thinking rarely fosters competition power struggle; rather it leads to social, economic, and political stagnation. People in such structure looks like contented with their rank, everything, and everyone stayed almost in the same position. Nothing changes the routine daily life. On the contrary, the hierarchical structure has graded levels, from the bottom to the top. That means many-valued logic represents this structure(as illustrated in fig.4).

Moreover, the hierarchical structure plays an important role in many situations that require rapid response, such as in the military field, but it does not fit all situations. Therefore, "Shepherd" criticized the bias for hierarchy -as an institution formed by men- in the social structures of science. Such bias is manifested in sciences classification according to their importance, or in authorship of papers by highlighting the lab chief's name or honoring him, while the technicians or the actual contributors may be given a subservient or less importance. (*Ibid*, pp. 125-126.)

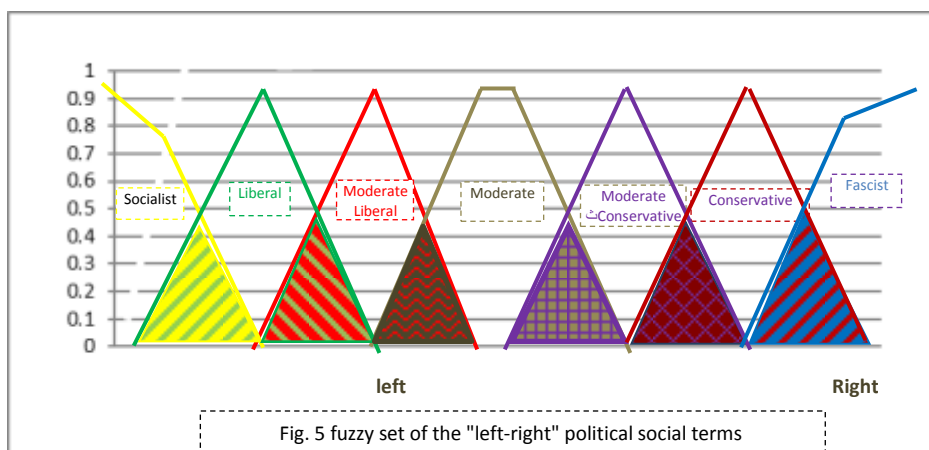
In her description, "Shepherd" has detected the exclusion nature in many scientific institutions. Hence, two questions may arise here. The first one is; how could the Western society achieve much progress if the scientific institutions are dominated by such hierarchical structure? The second one is;are such achievements possible if each person is dominated by the competition power struggle because of eager climbing to the summit?

Based upon shepherd's discussion of this structure, she aimed to emphasize that these achievements would be, scientifically and humanly, much more healthy and flexible. Therefore, she criticized the competitive power struggles in the scientific fields, and legalized it in military fields. As an alternative to such competition, she discusses how "healthy competition based on love can inspire creativity, augment motivation, foster diversity and flexibility, stimulate a striving for excellence, and push people to go beyond themselves." (*Ibid*, p. 189.) Without such love, or if competition undermines cooperation, it crosses over to a power motive and becomes corrosive and nasty. Consequently, both competitions are important, however, each of them has its own field. The healthy competition is more necessary in the scientific fields in order to stimulate a striving advancement, for individuals, communities, and societies too, while the competition power struggle maybe the most important in the military fields, as in the military maneuvers, for example.

On the contrary, the membership functions of the fuzzy set represent as piral structure, and interrelationship, on horizontal level; an openness and qualitative characteristics, on vertical level. Since the cyclic structure represented by the membership function of many-valued sets on the horizontal level, fosters sameness and repetition, it has disadvantage of hindering the advancement of individuals. It lacks interference between many levels. The spiral structure, on the other hand, embraces both multiplicity and advancement. Such structure represents fuzzy thinking which appeared in the overlapping of fuzzy subsets that manifests in institutional jobs (works). Such kind of institutions are based on cooperation and collaboration alternative to power struggle, on participation in decision-making alternative to impose order from above. This collaboration is enhanced by what is

called "quality circles" which can help employees to resolve their problems, and to improve productivity. The spiral structure allows conversations to take place on the same level, which lead to advancement on the multi levels. In this sense, the membership functions of fuzzy sets^(*) represent the interrelation, on the macro- and micro levels. It fosters the relations between subject and object, between philosophy and science, between intuition and inference, and between theoretical logic and real life. These relations, as characterized by the post-modern way of thinking, give fuzzy logic a liberated role from dualities and all related notions that have exclusion characters. In addition, the overlap between fuzzy subsets may interpreted as embodying the conversation between different contrary groups. Furthermore, smooth transition of the membership values from full membership to non-membership and vice versa, understood as flexible mentality, and open- minded.

Fuzziness gives us more choices. For example, fuzzy sets capture the segments of left-right spectrum and limit the agenda for political debate and thought. As illustrated in fig. 5, the fuzzy term may divided into seven or more fuzzy subsets. Far-left socialists overlap to some degree with liberals, and these overlap with moderates and so on out to far-right fascists. It means that people are liberal and not liberal to some degree, or moderate and moderate conservative to other degrees. Few of them are pure liberals or pure conservatives. Thus, it is rare that people belong only to one subset. (Bart Kosko, *the Fuzzy Future: From Society and Science to Heaven in a Chip*, pp. 29-30.)



In addition, fuzziness represents the qualitative characters of human and social phenomena. These characters are manifest in the fuzzy membership function. The degrees of membership may be so small or large that we take them as none or all, as zero or one. So the membership function drawn as a curve that approach the extremes of zero and one but need not reach them. (Kosko, *Fuzzy Thinking*, p. 136.) For example, as shown in fig. 5, even if someone is liberal to some degree, there may be another onemore or less liberal, more or less moderate, and more or less conservative.

^(*)Michael Smithson has grouped four strategies for assigning membership each of which is suited for specific research purposes, while "Jay Verkuilen" has considered three strategies for assigning them. c. f., Michael Smithson & Jay Verkuilen: *Fuzzy Set Theory: Applications in the Social Sciences*, pp. 21-25& Jay Verkuilen, "Assigning membership in a fuzzy set analysis", pp. 462-496.

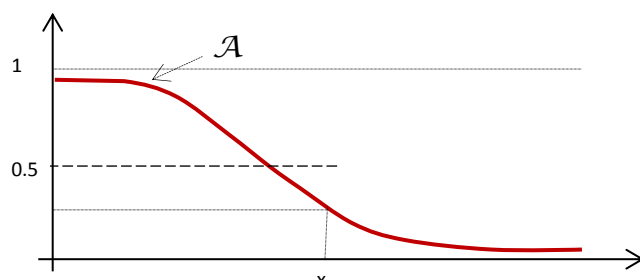


Fig. 6 fuzzy set membership function

In general, fully membership and non-membership values depend on the context and the purpose of the studying at hand. Thus, the fully membership is not at all absolute belonging. (As illustrated in fig. 6 fuzzy set membership function).

Looking attentively to these membership functions from their connection with feeling, intuition, and free will, we may understand the membership function of two-valued set, which represents the dualistic either/or thinking as it embraces both creative and destructive human activity. This membership function looks like such activity and like the human free will that transfers dramatically according to the importance of purpose. The dualistic either/or thinking is the warfare and destruction thinking, as well as the revolutionary movements that have no time to think. The revolutionary movement aims to decompose and negate the old situation, and recreate it. According to such meaning, dualistic either/or thinking is to have decisive principle attitude toward the actual world. In addition, it is a creative generative act that is accompanied with scientific and philosophical thinking; i.e. in the creation, ideas emerge unexpectedly, which looks like the abrupt step in the membership function from zero to one, or like transfers truth values from true to false, without any gradation.

In the same context, the membership function of many-valued set may understood as it represents both the swing feeling and multiplicity thinking. Therefore, many-valued logic looks like someone's emotion swung between two or more different feelings, or his opinion swung between two or more points of view. On the other hand, it is a kind of thinking that stimulates, somewhat, finding out the third, fourth or fifth excluded middle, exposes to diversity of opinions and multiple perspectives, regardless of possible fruitful conversation. This kind of conversation might be fruitless if there is a gap between opinions. Such gap is represented graphically by non-overlapping between many-valued subsets.

On the contrary, the membership functions of fuzzy sets look like smooth negotiation and thinking. It is peacemakers and calmness's thinking, as well as the logic of everyday routine life. In addition, fuzzy thinking equals the grayness that people almost have difficulties to be decisive toward many propositions that need such attitudes. This kind of grayness makes people as if they are careless or neutral. Nevertheless, fuzzy logic is a kind of thinking that always fosters to find out the excluded middle. In addition, it fosters the mutual conversation between different multi-groups. The creation in such thinking emerges as accumulated experiences, and continual hard working. Such continuity is represented logically by smooth transition of truth-values, from one to another value, which looks like as smooth as thinking, as smooth as transference and interference in real vague phenomena.

The overlapping between fuzzy subsets represents the anti-exclusion nature of fuzzy logic. Such overlapping reflects the polar relationship in human being's life and its

importance for them, and the complementarity between different sciences in order to achieve much progress, liberation, and openness.

Beside these philosophical and social significations of membership functions, we could find more significations through the basic operations on sets, i.e. intersection, union, complement, and the subset or inclusion between two sets, which fosters and detects more significations.

Basic operations on sets and their significations:

Crisp- and fuzzy sets use the same basic operations. However, there are many differences in each of them and in its consequences. Therefore, in crisp sets the intersection between two crisp sets ($A \cap B$) is a crisp set whose elements are common to (A) and (B), or are belonging necessarily to both of them. The union between two crisp sets ($A \cup B$) is a crisp set whose elements are either belong tot (A), or (B) including any element that belongs to both sets. The complement of the crisp set (A) denoted by (\bar{A}) is a set, which consists of all elements in the universal set (U) that are not in the given set. Finally, the inclusion between two crisp sets ($A \subseteq B$) or (A) is a subset of (B) if every element of (A) is also an element of (B) (Guanrong Chen & Trung Tat Pham: *Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems*, p. 2 & Bojadziew, *Fuzzy Logic for Business*, pp. 3-4.)

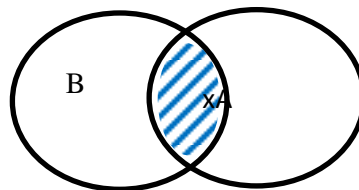


Fig. 7 $A \cap B$ Classical Intersection

As illustrated in (fig. 7) the intersection operation draws a sharp line between the members and non-members of the set. By the same way, complement operation (as in fig. 1) separates the members of the set from non-members, or from the members of its complement set. This means that these two operations foster separationism and exclusionism. According to the first operation, the members that are not belonging to both sets are excluded, and according to the second one, those who are not belonging to the given set are excluded, i.e. the member cannot join between belonging to the set and to its complement. On the other hand, the intersection operation detects precision in selecting the elements that fit the necessary conditions of such set, which may be interpreted as impartial selection.

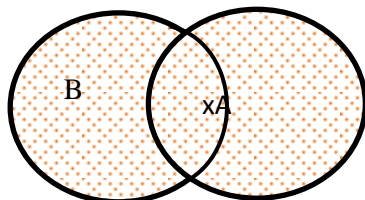


Fig. 8 $A \cup B$ Classical Union

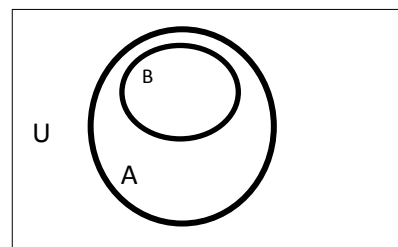
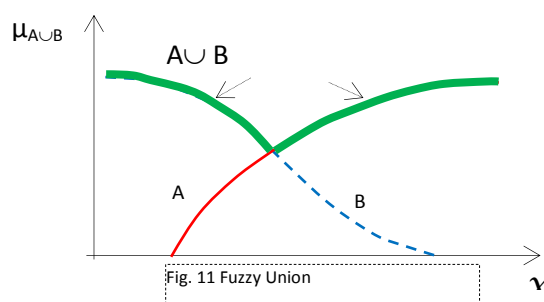
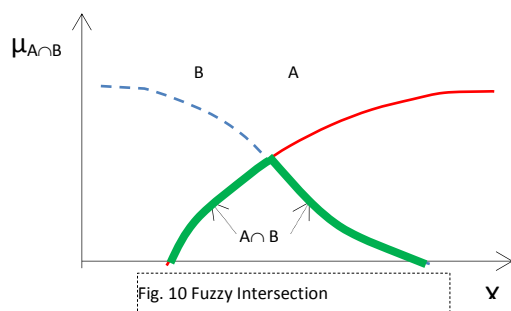
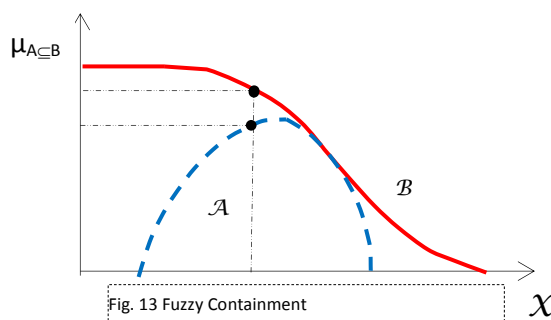


Fig. 9 $A \subseteq B$ Classical Containment

Similarly, the union and inclusion operations foster universalism. The result of the union operation between two crisp sets (fig. 8) is a set that consists of all members belonging, at least, to one of the sets. The inclusion between two crisp sets (fig. 9) is always that the whole contains its parts, or the set contains all its subsets to 100% degree. That means, the inclusion operation, or the subset relation, represents the dualistic either/or, all or none thinking, which is agreed with Aristotelian principle, but disagreed with fuzzy inclusion (Kosko, *Fuzzy Thinking*, pp. 55-58.)The union could be interpreted, in some cases, as flexibility in assigning sufficient conditions. In addition, the inclusion operation may interpreted as understanding and comprehension where patience and wisdom are needed.



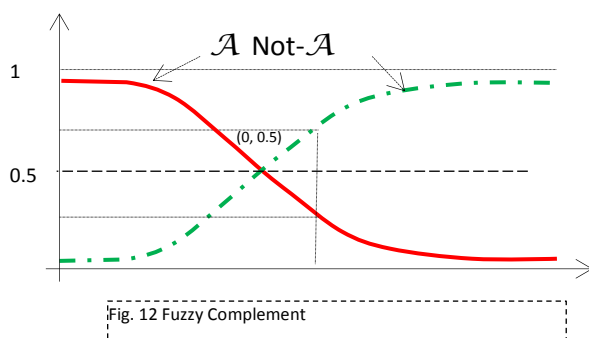
On the contrary, the intersection of two fuzzy sets (\mathcal{A}) and (\mathcal{B}) (fig. 10) is the largest fuzzy set, which is contained in both sets (i.e. the minimum membership score of each case in both sets). The union of (\mathcal{A}) and (\mathcal{B}) (fig. 11) is the smallest fuzzy set, which contains both sets (i.e. the maximum membership score of each case in both sets) (Zadeh, "Fuzzy Sets", pp. 340-1.)



The more operation that detects the differences between crisp- and fuzzy sets is the complementary. The complement of a fuzzy set (\mathcal{A}), as illustrated in (fig. 12), is a fuzzy set (not- \mathcal{A}) that each case's membership in a set and in the set's complement must sum to (1).The membership function of a set is symmetrical to the membership function of its complement with respect to the line $\mu=0.5$.(Bojadziev, *Fuzzy Logic for Business*, p. 16 & Kosko, *Fuzzy Thinking*, p. 136.)Therefore, each element belongs both to the fuzzy set and to the set's complement.

The fuzzy inclusion is more general than the classical one. As illustrated in (fig. 13), we consider that the fuzzy set (\mathcal{A}) is contained in the fuzzy set (\mathcal{B}),(or equivalently (\mathcal{A}) is a fuzzy subset of (\mathcal{B}), or (\mathcal{A}) is smaller than or equal to (\mathcal{B})), if and only if, for any element (x) in the universal set (\mathcal{U}),the membership value in (\mathcal{A}) is less than or equal

to its membership value in (\mathcal{B}). (Zadeh, "Fuzzy Sets", p. 340.) In that case, the fuzzy inclusion means that the part cannot totally contain the whole. However, it always partially contains it, i.e. the part contains the whole to some degree. (Kosko, *Fuzzy Thinking*, pp. 55-58.)



Suppose the element (x) has a membership scores in the fuzzy set (\mathcal{A}) and (\mathcal{B}) equal to [0.24], [0.3], respectively. Thus, the membership score of (x) in the intersection of (\mathcal{A}) and (\mathcal{B}) is the smallest value, which is equal to [0.24]; in the union is the largest value, which is equal to [0.3]; in the complement of (\mathcal{A}) is [0.76]; in the complement of (\mathcal{B}) is [0.70].

Looking attentively to these values, we can recognize that they represent the belongings of human beings in real life. While each person could belong partially to any set or community, s/he could belong, at the same time, to any set and to its complementary. In addition, the degrees of membership value are different from one person to another; I mean the individuals' belongings to religion, race, political party, and social communities, etc. are different from one person to another. Hence, the basic operations on fuzzy sets are anti-exclusionism, anti-separationism, and anti-universalism; rather they foster the between-ness and interrelationships. Thus, there is no reason to undermine any field of knowledge only if the criteria of objectivity do not satisfy; no reason to violate individual's freedom only if their belonging to thinking or a political party does not satisfy us, or only if s/he breaks the social norms. In addition, there is no reason to separate the different fields of knowledge because of specialty.

However, these basic operations could be interpreted in another way. The fuzzy intersection operation- as a minimum membership value- may signify, in some cases, as if someone accepts the minimum standard of things, i.e. as s/he is satisfied or contented in his life. That could be represented at two levels; the first one is at the individual level, as a limited ambitious, the second one is at the community's level, as an abandonment of some more important conditions. This limitation appears especially when the intersection is between a set and the set's complimentary; because the max-min value is no more than (0.5). By contrast, the union operation -as a maximum membership value- may signify, in some cases, as if it is an attempt to go beyond this limitation, but will be never accesses to the high level as the classical union does.

Consequently, according to two-valued logic, there are only binary absence-existence relations, i.e. either there is a relation or not. If such thinking is adopted by dictatorial regimes, then the binary relation would take a kind of up-down relation between the ruler and the people. That would make people surrendered and controlled. A kind of

scientific, thoughtful, social, and political stagnation would almost privilege the society. People of such society are as children as they obey the orders. They look like bodies without life, genius without egos, and brains without thinking. By contrast, if such thinking is adopted by democratic regimes, the binary relation would mean a highly organized way of life. Here, the precision of classification means the obligation of both citizens and leaders to rules.

The relations in many-valued logic are, somewhat, wider than those in two-valued logic. They include, besides the absence/existence relation, neutral or undefined one. However, unlike Linda Shepherd, I assert that many-valued logic represents the hierarchical structure. This kind of structure leads, in my viewpoint, in autocratic systems, to break the connection between people, or make untrusted social relations, by separating them into different levels. In addition, since there is only room at the top of the pyramid for the few elite, hierarchical structures usually foster a nasty competition, i.e. a power struggle. People are ruled by fear and intimidation. Each of them patterns his behavior according to that of his superior, observing and listening to those with higher status, all of them are as followers as they obey the orders of the top of the pyramid. Decisions are made from the top down, where the leader informs the followers how they should implement them. Therefore, this system needs a special leader (karizma), and makes others irresponsible, careless, and uninterested in developing their institutions. Consequently, all of these things will increase autocracy in the institutions. (Shepherd, *Lifting the Veil*, pp. 125-126, p. 138.) By contrast, such structure fosters, in democratic systems, a healthy competition, which leads to promote individuals and communities, and to develop societies as a whole.

Unlike binary and multiple relations, fuzzy ones are intertwined to varying degrees, and shaped in different kinds. As for human beings, the relation between two or more persons is neither represented by existence/absence binary relations, such as friendship-enemy, love-hate, and mutual benefits-contradict interests, nor is represented by existence-neutral-absence triple relations. Also, the relations between two or more countries does not take one form only, such as peace/war relation, rather they are interlocking with and overlapping between each other at many levels. Therefore, fuzzy thinking substitutes collaboration by struggle, dialogue by exclusion, and spiral dynamic structure by statistic or circular one. This allows us to say that if such thinking has prevailed among societies with democratic system, all citizens would have responsibility to their works and that would lead to scientific, economic, social, and political prosperity. Everyone would contribute to make more achievements, each one has an important role in society progress. On the contrary, if fuzzy thinking prevailed upon societies with autocratic system, a rioting, chaos, and disorder would be prevalent, confusion relationships would emerge in the whole society. For this sense, the rotation of spiral structure will not make developed social system, rather it will destroy it.

Conclusion

The purpose of the three logical systems are not the same. In formal logic, it is to keep mind away from fallacy, in symbolic logic, is to get rid of natural language's ambiguity, and in multi-valued logic, is to get rid of strictness of the formal logic principles. While, in fuzzy logic, the purpose is to avoid the overshoot and undershoot problems that resulted by traditional control, and to approach the characteristics of some human and social phenomena more efficiently and feasibility by specialists in this logic.

Although the purposes in these logical systems are different, each of them represents an aspect of real world, and reflects one way of thinking better than the other does.

Therefore, it is not true to say that one, but not the other, of these systems can represent real world, and reflect human thinking's ways, but each of them has the most suitable and appropriate scope to represent.

These logical systems, which are an organized expressions of some human thinking's ways, came later without entirely exhausting them. However, although the important role of these systems in theoretical and practical life to enrich and organize thinking ways, they should not become an instrument to restrict human thinking ways, rather human being should always seek to free from all existing logical systems, because human and social phenomena are more rich and diverse than these systems. In another words, these logical systems do not exhaust human beings' life.

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